chap 16, Ex 16A P397

(1) too easy

(2) $s = t^3 + t \Rightarrow v = \frac{ds}{dt} = 3t^2 + 1$

when $t = 3$, $v = 28$ m/s

(3) $s = 5t^2 - t^3 \Rightarrow v = \frac{ds}{dt} = 10t - 3t^2$

$\Rightarrow a = \frac{dv}{dt} = 10 - 6t$

at $t = 1$, $a = 4$ m/s^2

(4) /

(5) /

(6) $V = 4t + 5$ and $s = 10$ when $t = 1$

So $v = \frac{ds}{dt} = 4t + 5 \Rightarrow s = \int 4t + 5 \, dt$

$= 2t^2 + 5t + c$

But $s = 10$, $t = 1 \Rightarrow 10 = 2 + 5 + c \Rightarrow c = 3$

So $s = 2t^2 + 5t + 3$

Now, when $t = 2$, $s = 2(4) + 5(2) + 3 = 21$ m

(7) ✓

*(8) given $a = 6t$ and "initially at rest" $\Rightarrow t = 0, v = 0$

So $a = \frac{dv}{dt} = 6t \Rightarrow v = \int 6t\,dt$

$= 3t^2 + c$

But $t = 0, v = 0$ so $0 = 0 + c \Rightarrow c = 0$

$\therefore v = 3t^2$.

When $t = 4$, $v = 3(16) = 48\,m/s$

*(9) given $a = \frac{2}{3}t$, $s = 10$ & $v = 4$ when $t = 3$,

we have $a = \frac{dv}{dt} = \frac{2}{3}t$

So $v = \int \frac{2}{3}t\,dt = \frac{1}{3}t^2 + c$

But $v = 4$ when $t = 3$ so $4 = \frac{1}{3}\cdot 9 + c \Rightarrow c = 1$

$\therefore v = \frac{1}{3}t^2 + 1$

Now $v = \frac{ds}{dt} = \frac{1}{3}t^2 + 1 \Rightarrow s = \int \frac{1}{3}t^2 + 1\,dt$

$\therefore s = \frac{1}{9}t^3 + t + c$

But $s = 10$ when $t = 3$, so $10 = 3 + 3 + c \Rightarrow c = 4$

So $s = \frac{1}{9}t^3 + t + 4$; Then $s(6) = \frac{1}{9}\cdot 6^3 + 6 + 4 = 34\,m$

(10) given $s = t^2 - 3$ Then

when $t = 2$, $s = +1$ m

$$v = \frac{ds}{dt} = 2t \ (m/s)$$

so $v(2) = 4$ m/s

If $v = 8$ Then $8 = 2t \Rightarrow t = 4$ secs

Then $t = 4$ when $v = 8$ so $s(4) = 4^2 - 3 = 13$ m

* (11) given $s = 2t^3 - 21t^2 + 60t$

a) ✓ b) $v = \frac{ds}{dt} = 6t^2 - 42t + 60$

body at rest $\Rightarrow v = 0$

so $0 = 6t^2 - 42t + 60$

$\Rightarrow t^2 - 7t + 10 = 0$

$$\Rightarrow t = \frac{7 \pm \sqrt{49 - 40}}{2} = \frac{7 \pm 3}{2} = 5, 2 \text{ secs}$$

c) initial velocity is when $t = 0 \Rightarrow v = 60$ m/s

d) $a = \frac{dv}{dt} = 12t - 42$

e) initial $\underline{a}$ is when $t = 0 \Rightarrow a = -42 \ m/s^2$

i.e a deccelaration

* (12) given $v = 8t - 3t^2$

a) ✓ b) $a = \frac{dv}{dt} = 8 - 6t$

c) at $t = 3$, $a = -10\ m/s^2$

d) $v = 8t - 3t^2 = \frac{ds}{dt} \Rightarrow s = \int 8t - 3t^2\, dt$

$= 4t^2 - t^3 + c$

But body is initially at 0 $\Rightarrow t = 0$ & $s = 0$

So $c = 0 \Rightarrow s = 4t^2 - t^3$.

So when $t = 3$, $s = 4(9) - 27 = 9m$

(13) ✓

(14) given $a = 2t$ & $v = 0$ when $t = 0$ (initially at Rest)

So $a = \frac{dv}{dt} \Rightarrow v = \int 2t\, dt$

$= t^2 + c$

But $v = 0$ when $t = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$

So $v = t^2$. Then when $t = 3$, $v = 9\ m/s$

For distance, $v = \frac{ds}{dt} = t^2$ so $s = \int t^2\, dt = \frac{1}{3}t^3 + c$

now, when $t = 0$, $s = 0 \Rightarrow c = 0$, so $s = \frac{1}{3}t^3$

So when $t = 3$, $s = \frac{1}{3}(27) = 9m$.

(15) ✓

(16) given $\underline{s} = t^3\underline{i} + 9t\underline{j}$

So $\underline{v} = \frac{ds}{dt} = 3t^2\underline{i} + 9\underline{j}$

when $t=2$, $\underline{v} = 3(4)\underline{i} + 9\underline{j}$ so $|\underline{v}| = \text{speed} = \sqrt{12^2+9^2}$
$= 15 \text{ m/s}$

(17) given $\underline{v} = 2t^2\underline{i} + 6\underline{j}$ Then

$\underline{a} = \frac{dv}{dt} = (4t)\,\underline{i}$

So when $t=3$, $\underline{a} = 12\underline{i}$ m/s^2

(*) (18) given $\underline{v} = 3t^2\underline{i} + 10t\underline{j}$

Then $\underline{v} = \frac{ds}{dt} \Rightarrow \underline{s} = \int 3t^2\underline{i} + 10t\underline{j}\, dt$
$= t^3\underline{i} + 5t^2\underline{j} + \underline{c}$

But $\underline{s_0} = (4\underline{i} - 4\underline{j})$ m at $t=0$

So $4\underline{i} - 4\underline{j} = 0 + 0 + \underline{c}$

So $\underline{s} = t^3\underline{i} + 5t^2\underline{j} + 4\underline{i} - 4\underline{j} = (t^3-4)\underline{i} + (5t^2-4)\underline{j}$

When $t=2$, $\underline{s} = (4\underline{i} + 16\underline{j})$ m.

But $\underline{s_0} = (4\underline{i} - 4\underline{j})$ so location of $\underline{s}$ from $\underline{s_0}$ is $\underline{s} - \underline{s_0}$ i.e

$\underline{s} - \underline{s_0} = (4\underline{i} + 16\underline{j}) - (4\underline{i} - 4\underline{j}) = 20\underline{j}$, $\therefore |\underline{s} - \underline{s_0}| = 20$ m

(19) given $\underline{a} = (4t+2)\underline{i} - 3\underline{j}$ & $\underline{v} = 0$ when $t = 0$

Then $\underline{a} = \frac{dv}{dt} \Rightarrow \underline{v} = \int (4t+2)\underline{i} - 3\underline{j}\, dt$

$= (2t^2 + 2t)\underline{i} - 3t\underline{j} + c$

But $0\underline{i} + 0\underline{j} = 0\underline{i} - 0\underline{j} + \underline{c}$ so $\underline{c} = 0\underline{i} + 0\underline{j}$

$\therefore \underline{v} = (2t^2 + 2t)\underline{i} - 3t\underline{j}$

when $t = 3$, $\underline{v} = (24\underline{i} - 9\underline{j})$ m/s

(20) given $\underline{a} = 6t\underline{i} + 2\underline{j}$

Then $\underline{a} = \frac{dv}{dt} = 6t\underline{i} + 2\underline{j} \Rightarrow \underline{v} = \int 6t\underline{i} + 2\underline{j}\, dt$

$= 3t^2\underline{i} + 2t\underline{j} + \underline{c}$

But given $\underline{v} = 4\underline{i} - \underline{j}$ when $t = 1$, Then

$4\underline{i} - \underline{j} = 3\underline{i} + 2\underline{j} + \underline{c}$

$\Rightarrow \underline{c} = \underline{i} - 3\underline{j}$. So $\underline{v} = 3t^2\underline{i} + 2t\underline{j} + \underline{i} - 3\underline{j}$

$= (3t^2 + 1)\underline{i} + (2t - 3)\underline{j}$

Now, $\underline{v} = \frac{ds}{dt}$ so $\underline{s} = \int (3t^2 + 1)\underline{i} + (2t - 3)\underline{j}\, dt$

$= (t^3 + t)\underline{i} + (t^2 - 3t)\underline{j} + \underline{c}$

But given $\underline{s} = 2\underline{i} + 3\underline{j}$ when $t = 1$ Then

$$2\underline{i} + 3\underline{j} = 2\underline{i} - 2\underline{j} + \underline{c} \Rightarrow \underline{c} = 5\underline{j}$$

So $\underline{s} = (t^3 + t)\underline{i} + (t^2 - 3t)\underline{j} + 5\underline{j}$

So when $t = 3$, $\underline{s} = (30\underline{i} + 5\underline{j})$ m

(21) Given $\underline{s} = (2\sin t)\underline{i} + (3\cos t)\underline{j}$

Then $\underline{v} = \dfrac{d\underline{s}}{dt} = (2\cos t)\underline{i} - (3\sin t)\underline{j}$

& $\underline{a} = \dfrac{d\underline{v}}{dt} = (-2\sin t)\underline{i} - (3\cos t)\underline{j}$.

when $t = \frac{\pi}{2}$, $\underline{a} = (-2\underline{i})$ m/s^2

(22) ✓

(23) ✓

(24) Given $\underline{v} = (4\cos 2t)\underline{i} + (2\sin 2t)\underline{j}$

Then $\underline{v} = \dfrac{d\underline{s}}{dt} \Rightarrow \underline{s} = \int (4\cos 2t)\underline{i} + (2\sin 2t)\underline{j} \, dt$

using integration by substitution ($u = 2t$ ....) we obtain

$$\underline{s} = (2\sin 2t)\underline{i} - (\cos 2t)\underline{j} + \underline{c}$$

given $\underline{s} = 6\underline{i} - 2\underline{j}$ when $t = \frac{\pi}{4}$ Then

$$6\underline{i} - 2\underline{j} = 2\underline{i} - 0\underline{j} + \underline{c} \Rightarrow \underline{c} = 4\underline{i} - 2\underline{j}$$

So $\underline{s} = (2\sin 2t + 4)\underline{i} - (\cos 2t + 2)\underline{j}$

So when $t = \pi$, $\underline{s} = 4\underline{i} - 3\underline{j}$

So distance is $\sqrt{4^2 + 3^2} = 5\text{m}$

(25) /

(26) given $\underline{a} = (6\sin 6t)\underline{i} + (9\cos 3t)\underline{j}$ Then

$$\underline{a} = \frac{d\underline{v}}{dt} \Rightarrow \underline{v} = \int (6\sin 6t)\underline{i} + (9\cos 3t)\underline{j}\, dt$$

$$= (-\cos 6t)\underline{i} + (3\sin 3t)\underline{j} + \underline{c}$$

(use integration by substitution to obtain This answer)

given $\underline{v} = \underline{i} + 3\underline{j}$ when $t = \pi/6$ we have

$$\underline{i} + 3\underline{j} = \underline{i} + 3\underline{j} + \underline{c} \Rightarrow \underline{c} = 0\underline{i} + 0\underline{j}$$

So $\underline{v} = (-\cos 6t)\underline{i} + (3\sin 3t)\underline{j}$

Now $\underline{v} = \frac{d\underline{s}}{dt} \Rightarrow \underline{s} = \int (-\cos 6t)\underline{i} + (3\sin 3t)\underline{j}\, dt$

$= (-\frac{1}{6}\sin 6t)\underline{i} - (\cos 3t)\underline{j} + \underline{c}$

(... again by integration by substitution)

given $\underline{s} = 5\underline{i} + 2\underline{j}$ when $t = \frac{\pi}{6}$ Then

$5\underline{i} + 2\underline{j} = 0\underline{i} + 0\underline{j} + \underline{c}$

So $\underline{s} = (-\frac{1}{6}\sin 6t + 5)\underline{i} + (-\cos 3t + 2)\underline{j}$.

when $t = \frac{\pi}{3}$, $\underline{s} = (5\underline{i} + 3\underline{j})$ m

27 given $\underline{s} = \begin{pmatrix} t^2 - 5 \\ t^2 - 3t + 2 \end{pmatrix}$

a) Particle crosses x-axis when $y = 0$. So

$t^3 - 3t + 2 = 0 \Rightarrow (t-1)(t-2) = 0$

$\therefore$ $t = 1$ sec or 2 sec.

b) $\underline{v} = \frac{d\underline{s}}{dt} = \begin{pmatrix} 2t \\ 2t - 3 \end{pmatrix}$

So when $t = 6$, $\underline{v} = \begin{pmatrix} 12 \\ 9 \end{pmatrix}$, so speed $= |\underline{v}|$

$= \sqrt{12^2 + 9^2}$

$= 15$ m/s

c) $\underline{a} = \frac{d\underline{v}}{dt} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2\underline{i} + 2\underline{j}$; Then $\underline{F} = m\underline{a}$ implies

$\underline{F} = 6(2\underline{i} + 2\underline{j}) = (12\underline{i} + 12\underline{j})$ N

(28) / Same principle as (27)

(29) given

$$\underline{s} = \begin{pmatrix} t^3 - t^2 - 4t + 3 \\ t^3 - 2t^2 + 3t - 7 \end{pmatrix}$$

and mass = 4 kg Then

a) For Particle to cross The line $y = x$ we have

$$t^3 - t^2 - 4t + 3 = t^3 - 2t^2 + 3t - 7$$

$\therefore \quad t^2 - 7t + 10 = 0 \quad \Rightarrow \quad (t-5)(t-2) = 0$

So $t =$ 5 secs or 2 secs

b) $\underline{v} = \frac{d\underline{s}}{dt} = \begin{pmatrix} 3t^2 - 2t - 4 \\ 3t^2 - 4t + 3 \end{pmatrix}$, so when $t = 4$ we have

$$\underline{v} = \begin{pmatrix} 36 \\ 35 \end{pmatrix} = (36\underline{i} + 35\underline{j}) \text{ m/s}$$

c) $\underline{a} = \frac{dv}{dt} = \begin{pmatrix} 6t - 2 \\ 6t - 4 \end{pmatrix}$; given $t = \frac{2}{3}$ we have

$$\underline{a} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

given $m = 4$ kg, $\underline{F} = m\underline{a} \Rightarrow \underline{F} = 4\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ N

(30) / Same principle as (29)

(31) given $\quad \underline{s} = \begin{pmatrix} 2 - \cos 3t \\ 6 \sin 2t \end{pmatrix}$

a) if $\underline{s}$ never crosses y-axis, x term can never be 0. So set x term to 0 & find a contradiction.

$$\therefore \quad 2 - \cos 3t = 0 \Rightarrow \cos 3t = 2$$

But cos of any angle is always in range $[-1, 1]$
So can never become 2. Hence particle never crosses y-axis

b) $\underline{v} = \frac{d\underline{s}}{dt} = \begin{pmatrix} 3 \sin 3t \\ 12 \cos 2t \end{pmatrix}$. At $t = \frac{\pi}{6}$ we have

$$\underline{v} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \text{ m/s}$$

c) $\underline{F} = m\underline{a}$, where $\underline{a} = \frac{d\underline{v}}{dt} = \begin{pmatrix} 9 \cos 3t \\ -24 \sin 2t \end{pmatrix}$.

At $t = \pi$, $\underline{a} = \begin{pmatrix} -9 \\ 0 \end{pmatrix}$, so $\underline{F} = 2 \begin{pmatrix} -9 \\ 0 \end{pmatrix} = \begin{pmatrix} -18 \\ 0 \end{pmatrix} N$

(for $m = 2$).

(32) given

$$\underline{s} = \begin{pmatrix} 2\sin t + \sin 2t \\ 4\cos t + \cos 2t \end{pmatrix}$$

a) $\underline{v} = \frac{d\underline{s}}{dt} = \begin{pmatrix} 2\cos t + 2\cos 2t \\ -4\sin t - 2\sin 2t \end{pmatrix}$

So when $t = \frac{\pi}{3}$, $\underline{v} = \begin{pmatrix} 0 \\ -3\sqrt{3} \end{pmatrix}$ m/s

b) $\underline{a} = \frac{d\underline{v}}{dt} = \begin{pmatrix} -2\sin t - 4\sin 2t \\ -4\cos t - 4\cos 2t \end{pmatrix}$

So when $t = \frac{\pi}{2}$, $\underline{a} = \begin{pmatrix} -2 \\ +4 \end{pmatrix}$ m/s^2

c) At $t = \pi/3$, $\underline{a} = \begin{pmatrix} -3\sqrt{3} \\ 0 \end{pmatrix}$ m/s^2

Now, $\underline{F} = m\underline{a}$ $\Rightarrow$ gradient of $\underline{F}$ is same as gradient of $\underline{a}$

So gradient of $\underline{a}$ must be flat $\Rightarrow \theta = 0$.

Test: $\theta = \tan^{-1}\frac{y}{x} \Rightarrow \theta = \tan^{-1}\frac{0}{-3\sqrt{3}} = 0°$

So force is // to x-axis

(33) – (43) : /

(47) given $a = \frac{3t^2}{2}$ Then

$$a = \frac{dv}{dt} = \frac{3t^2}{2} \Rightarrow v = \int \frac{3t^2}{2}\, dt$$
$$= \frac{t^3}{2} + c$$

But at O, $v = 1$ when $t = 2 \Rightarrow 1 = \frac{8}{2} + c \Rightarrow c = -3$

So $v = \frac{t^3}{2} - 3$

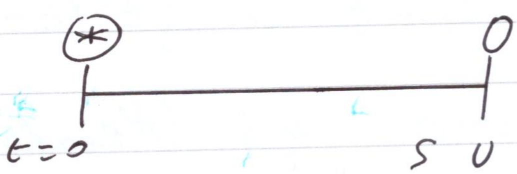
So at $t = 4$, $v = \frac{4^3}{2} - 3 = 29$ m/s

Then

$$v = \frac{ds}{dt} = \frac{t^3}{2} - 3 \quad \text{So}$$

$$s = \int \frac{t^3}{2} - 3\, dt = \frac{1}{4}\frac{t^4}{2} - 3t + c$$

Now consider the diagram below



S	U	V	A	T
		1		2
		?		4
0				2

} done

$S = 0$ at $t = 2$ (see question). So

$$0 = \frac{1}{4}\frac{2^4}{2} - 3(2) + c \Rightarrow c = 4$$

So distance from O after $t=4$ secs is distance from ⊛ minus distance ⊛O. I.e. $S(4) - S(2)$

So $\quad$ distance $= \frac{1}{4}\frac{4^4}{2} - 3(4) + 4 - \left(\frac{1}{4}\frac{2^4}{2} - 3(2) + 4\right)$

$= 24 - 0 = 24m$

(45) given $a = 2t$ Then $a = \frac{dv}{dt} = 2t \Rightarrow v = \int 2t\,dt$.

So $v = t^2 + c$. But $v=0$ when $t=0 \Rightarrow c=0$

So $v = t^2$.

Then $\quad v = \frac{ds}{dt} = t^2 \Rightarrow s = \int t^2\,dt = \frac{t^3}{3} + c$

But $s=0$ when $t=0 \Rightarrow c=0$.

So $\quad s = \frac{t^3}{3}$.

For $s=v$ we have $\frac{t^3}{3} = t^2 \Rightarrow \frac{1}{3}t^2(t-3)=0$

$\Rightarrow \frac{1}{3}t^2 = 0 \quad \Rightarrow t=0$ $\qquad$ (Already given)

& $\quad t-3=0 \Rightarrow t = 3$ secs.